Amount of Reward Has Opposite Effects on the Discounting of Delayed and Probabilistic Outcomes

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Previous research has shown that the value of large future rewards is discounted less steeply than is the value of small future rewards. These experiments extended this line of research to probabilistic rewards. Two experiments replicated the standard findings for delayed rewards but demonstrated that amount has an opposite effect on the discounting of probabilistic rewards. That is, large probabilistic amounts were discounted at the same or higher rates than small amounts. Although amount had opposite effects on the discounting of delayed and probabilistic rewards, nevertheless, the same form of mathematical function accurately described discounting of both types of reward. The findings suggest that fundamentally similar, but not identical, processes are involved in decision making regarding delayed and probabilistic rewards. The implications of these findings for impulsivity and self-control are discussed.

People frequently encounter situations in which they have to choose between two outcomes that differ in both magnitude and delay. For example, one may have to choose the amount of money to be deducted from one's paycheck and invested in a retirement account. In this situation, the choice is either having more money available now and less later or having less available now but a much larger amount at a later time. People also frequently encounter situations in which they have to choose between two outcomes that differ in their probability as well as their magnitude. For example, the choice might be between two investment opportunities, one of which is riskier but has a higher potential payoff, whereas the other is safer but has a lower rate of return. This research concerns the question of whether similar decision-making processes are involved in both kinds of choices, those involving delayed and those involving probabilistic outcomes.

It is intuitively obvious that the subjective value of an outcome decreases as the time until its occurrence increases and that subjective value also decreases as the odds against an outcome increase. People would generally prefer to receive $100 now rather than in a month and would prefer $100 for sure rather than a 1-in-10 chance of receiving the same amount. One approach to understanding the processes involved in the discounting of delayed and probabilistic outcomes involves attempting to find a mathematical model that accurately describes the observed changes in subjective value (e.g., Myerson & Green, 1995; Rachlin, Raineri, & Cross, 1991). Previous research on the discounting of delayed outcomes has shown that preference between immediate and delayed rewards is well described by a hyperbola-like function of the form,

\[ V = \frac{A}{1 + kD}, \]  

where \( V \) is the present, subjective value of a reward of amount \( A \), \( D \) is the time until its receipt, and the parameter \( k \) governs the rate at which subjective value decreases. With respect to the parameter \( s \), when its value is 1.0, Equation 1 is a simple hyperbola (Mazur, 1987; Rachlin, 1989); when its value is less than 1.0, as is frequently observed, it means that (for the same value of \( k \)) discounting becomes less steep as the delay gets large to a greater extent than would be true for a simple hyperbola (Green, Fry, & Myerson, 1994; Myerson & Green, 1995).

Rachlin et al. (1991) reported that the discounting of probabilistic rewards can be described by a simple hyperbola,

\[ V = \frac{A}{1 + h\Theta}, \]  

where \( \Theta \) represents the odds against receipt of a probabilistic reward and \( h \) is a parameter (analogous to \( k \) in Equation 1) that reflects the rate of decrease in subjective value. (It should be noted that \( \Theta = (1 - p)/p \) where \( p \) is the probability of receipt.) Rachlin's derivation of this function assumes that the discounting of delayed rewards is fundamental. He assumes that individuals treat probabilistic rewards as if they were repeated gambles and that the subjective value of a gamble is determined by the average waiting time until a win (i.e., multiplying \( \Theta \) by the interval between gambles provides an estimate of the average waiting time for the probabilistic reward).
If the discounting of delayed rewards is the fundamental process underlying decisions involving probabilistic rewards, then the same mathematical model that describes changes in subjective value as a function of delay should also describe changes in subjective value as a function of the odds against receiving a reward. Thus, if the discounting of delayed rewards is better described by a model like Equation 1, which includes an exponent, than it is by a simple hyperbola, then the discounting of probabilistic rewards likewise should be better described by a model that includes an exponent:

$$ V = A/(1 + h\theta)^t. \quad (3) $$

Although Rachlin has suggested that the discounting of delayed outcomes underlies decisions involving probabilistic outcomes (Rachlin et al., 1991), others have suggested that the discounting of probabilistic outcomes underlies decisions involving delayed outcomes (e.g., Green & Myerson, 1996; Stevenson, 1986). Regardless of which type of discounting is assumed to be more fundamental, if one form of discounting underlies both types of decisions, then the same form of mathematical model should describe both. However, the converse is not true. That is, even if both kinds of discounting are described by the same form of mathematical function, it does not necessarily follow that they both involve identical decision-making processes.

Therefore, the purpose of the present study was twofold. The first was to examine whether the same mathematical model would describe discounting of both delayed and probabilistic rewards. The second was to determine whether both types of discounting would be affected similarly by experimental manipulations. If they were not similarly affected, then even if they were described by the same form of model, such a result would raise serious questions as to whether the same processes were underlying both types of decisions. More specifically, in the present study, we examined whether variations in the amount of reward would differentially affect the discounting of delayed and probabilistic rewards. Previous research has established that the rate at which delayed rewards are discounted decreases with the amount of reward (e.g., Chapman & Winquist, 1998; Green, Myerson, & McFadden, 1997; Kirby & Marakovic, 1996; Myerson & Green, 1995; Raineri & Rachlin, 1993). In the present study, we examined whether a similar effect exists for the effect of amount on the discounting of probabilistic rewards.

### Experiment 1

**Method**

**Participants.** Sixty-eight students in the Faculty of Psychology at the University of Warsaw (Poland) participated in the experiment. There were 42 female and 26 male participants, ranging in age from 18 to 39 years ($M = 21.6, SD = 4.05$).

**Procedure.** The procedure was adapted for computer administration from that used in studies by Rachlin et al. (1991) and Green et al. (1994). Each participant was tested individually in a quiet room. Following instructions and practice trials, participants made a series of choices regarding hypothetical amounts of money displayed on a computer monitor. In the delayed reward conditions, choices were between an amount available immediately and another amount available after a delay. The two amounts and their respective delays (e.g., $10 now and $100 in 1 year) were displayed on the left and right sides of the screen. Similarly, in the probabilistic reward conditions, choices were between an amount available for sure and another amount available with a stated probability. The two amounts together with information about the probabilities of receiving the rewards (e.g., $10 for sure and a 30% chance of receiving $100) were displayed on the left and right sides of the screen. Participants indicated their choices by pressing one of two keys: the $ for the immediate (or certain) reward and the $ for the delayed (or probabilistic) reward.

The amounts of the delayed and probabilistic rewards were $500 and $10,000. Each of these amounts was presented at each of six delays (1 month, 6 months, 1 year, 3 years, 5 years, and 10 years) and at each of six probabilities (5%, 10%, 40%, 70%, 90%, and 95% chance). When the amount of the delayed or probabilistic reward was $500, 24 values of the immediate or certain rewards were used, ranging from $1 to $499. When the amount of the delayed or probabilistic reward was $10,000, the 24 values of the immediate or certain rewards ranged from $10 to $9,990.

Each participant was studied with each amount ($500 and $10,000) and type of reward (delayed and probabilistic) presented at all six delays and at all six probabilities. These delays and probabilities were always presented in order from the briefest to the longest delay (i.e., 1 month to 10 years) and from the highest to the lowest probability (i.e., 95% to 5% chance). In addition, each participant was studied with the different amounts of the immediate, certain alternatives presented in both ascending and descending order. Participants first chose between a specific amount ($500 or $10,000) of the delayed (or probabilistic) reward and immediate (or certain) reward, beginning with either the highest (descending series) or lowest (ascending series) of the 24 amounts. In the descending series, the amount of the immediate, certain reward was decreased successively until the participant changed his or her preference from the immediate, certain reward to the delayed (or probabilistic) reward. Conversely, in the ascending series, the amount of the immediate, certain reward was increased successively until the participant changed his or her preference from the delayed (or probabilistic) reward to the immediate, certain reward.

After a participant had been studied with both the $500 and $10,000 rewards at every delay and probability, with the immediate, certain amounts presented in either the ascending or descending order, he or she was tested again on the full sequence of choices, this time with the amounts of the immediate, certain rewards presented in the opposite order to that used the first time. The amount ($500 or $10,000) and type of reward (probabilistic or delayed), as well as the type of sequence (ascending or descending) of the immediate, certain alternative, were counterbalanced across subjects.

The amount of the immediate, certain reward judged to be subjectively equivalent to both the $500 and $10,000 rewards was determined for each participant at each delay and probability as follows. The subjective value was calculated as the average of two values at which the participant switched preference from the immediate, certain reward to the delayed or probabilistic reward on the descending series and the value at which the participant switched preference from the delayed or probabilistic reward to the immediate, certain reward on the ascending series. Thus, for both the $500 and $10,000 delayed rewards, six subjectively equivalent amounts were calculated, one for each of the six delays. Similarly, for both the $500 and $10,000 probabilistic rewards, six subjectively equivalent amounts were calculated, one for each of the six
probabilistic reward. Both reward amounts were fit simultaneously with separate parameters and a single \( k \) parameter differed as a function of the amount of the delayed or probabilistic reward. In both cases, the value of the \( s \) parameter was significantly less than 1.0: for delayed rewards, \( s = 0.75, t(9) = 2.94, p < .05 \); for probabilistic rewards, \( s = 0.56, t(9) = 10.77, p < .01 \). This finding argues for the inclusion of an exponent in both discounting functions. Fit simultaneously to the subjective values of the $500 and $10,000 delayed rewards (top panel), Equation 1 accounted for 99.1% of the variance. For probabilistic rewards (bottom panel), Equation 3 accounted for 99.4% of the variance when fit simultaneously to the subjective values of both reward amounts.

As may be seen in the top panel of Figure 1, when the rewards were delayed, the data points representing the subjective value of the smaller reward (expressed as a proportion) fell below those representing the subjective value of the larger reward. This was reflected in the estimates for the discounting rate parameters in Equation 1. The value of \( k \) for $500 was 0.167, whereas the value of \( k \) for $10,000 was 0.073, \( t(9) = 3.79, p < .01 \), indicating that the smaller reward was discounted at a significantly higher rate than the larger reward. In contrast, when the rewards were probabilistic, the data points representing the subjective value of the smaller reward (expressed as a proportion) fell above those representing the subjective value of the larger reward. This was reflected in the estimates for the discounting rate parameters in Equation 3. The value of \( h \) for $500 was 1.04, whereas the value of \( h \) for $10,000 was 0.75, \( t(9) = 2.94, p < .01 \). This finding argues for the inclusion of an exponent in both discounting functions. Fit simultaneously to the subjective values of the $500 and $10,000 delayed rewards (top panel), Equation 1 accounted for 99.1% of the variance. For probabilistic rewards (bottom panel), Equation 3 accounted for 99.4% of the variance when fit simultaneously to the subjective values of both reward amounts.

Results

To compare the discounting of the smaller ($500) and larger ($10,000) rewards, we expressed the subjective values of these rewards as a proportion of their nominal amount. For example, a delayed $500 reward that was judged equivalent in value to an immediate $100 reward had a subjective value of .20. After converting amounts to proportions, we used multiple nonlinear regression analysis to determine whether the discounting rate parameters (i.e., \( k \) and \( h \) for delayed and probabilistic rewards, respectively) differed as a function of the amount of the delayed or probabilistic reward. Both reward amounts were fit simultaneously with separate \( k \) parameters and a single \( s \) parameter (Myerson & Green, 1995). Such analyses were conducted on the individual data, as well as on the group median subjective values (Green et al., 1994).

Figure 1 shows the median subjective values of the $500 (circles) and $10,000 (triangles) rewards. (The inset shows a more detailed view of the data when the odds against receiving a reward were low.) The curved lines represent the best fitting discounting functions. Hyperbola-like functions of the same mathematical form (Equations 1 and 3) accurately described the discounting of both delayed and probabilistic rewards. In both cases, the value of the \( s \) parameter was significantly less than 1.0: for delayed rewards, \( s = 0.75, t(9) = 2.94, p < .05 \); for probabilistic rewards, \( s = 0.56, t(9) = 10.77, p < .01 \). This finding argues for the inclusion of an exponent in both discounting functions. Fit simultaneously to the subjective values of the $500 and $10,000 delayed rewards (top panel), Equation 1 accounted for 99.1% of the variance. For probabilistic rewards (bottom panel), Equation 3 accounted for 99.4% of the variance when fit simultaneously to the subjective values of both reward amounts.

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Figure 1. Subjective value as a function of delay until receiving a reward (top panel) and odds against receiving a reward (bottom panel). Data are group medians from Experiment 1 and represent the amount of an immediate, certain reward judged equal in subjective value to a delayed or probabilistic reward. The curved lines are the best fitting, hyperbola-like discounting functions (Equation 1 for delayed rewards and Equation 3 for probabilistic rewards). The inset shows a more detailed view of the data when the odds against receiving a reward were low.
was 6.56, whereas the value of \( h \) for $10,000 was 10.84, \( t(9) = 3.77, p < .01 \), indicating that the smaller probabilistic reward was discounted at a significantly lower rate than the larger probabilistic reward.

In addition to these group-level analyses, Equations 1 and 3 were also fit to the subjectively equivalent values for the delayed and probabilistic rewards from each individual participant. Figure 2 presents the results for the individual (i.e., P-31) whose discounting of delayed and probabilistic rewards was best fit by Equations 1 and 3 (top panels), as well as the results for the participants whose \( R^2 \) values for both delayed and probabilistic discounting were close to the group’s 75th, 50th, and 25th percentiles (P-36, P-66, and P-56, respectively). As may be seen, for the delayed rewards (left panels), the data points for the smaller ($500) amount tended to fall below the points for the larger ($10,000) amount, whereas for the probabilistic rewards (right panels), the data points for the smaller amount tended to fall above the points for the larger amount.

Statistical tests were conducted on the estimated values of the parameters of Equations 1 and 3 for individual participants. For delayed rewards, 56 of the participants had larger \( k \) values for the smaller amount, significantly more than expected by chance (Wilcoxon’s matched-pairs signed-ranks test, \( z = 4.90, p < .0001 \)), whereas for probabilistic rewards, 44 of the participants showed the opposite pattern. That is, they had smaller \( h \) values for the smaller amount, again significantly more than expected by chance (\( z = 3.02, p < .005 \)). Moreover, for 34 of the participants the smaller amount was associated with both a larger \( k \) and a smaller \( h \), significantly more than the number (i.e., 17) expected by chance (binomial test, \( z = 4.62, p < .0001 \)). Finally, 45 participants had estimated values of the exponent, \( s \), less than 1.0 for delayed rewards, and 59 had estimated values less than 1.0 for the probabilistic rewards, significantly more than expected by chance in both cases (binomial test, \( z = 2.55, p < .01 \), and \( z = 5.94, p < .0001 \), respectively).

**Discussion**

The results of Experiment 1 demonstrate that the same mathematical model (i.e., the isomorphic hyperbola-like functions Equations 1 and 3) accurately describes the discounting of both delayed and probabilistic rewards. In both cases, the exponent was significantly less than 1.0. The present findings are consistent with previous results obtained in our laboratory, showing that the addition of an exponent to a simple hyperbolic discounting function (such as Equation 2) significantly improves the fit to discounting data and that this is true for both delayed and probabilistic rewards (Green et al., 1994; Ostaszewski, Green, & Myerson, 1998). In addition, the results of Experiment 1 suggest that amount has opposite effects on the discounting of delayed and probabilistic rewards. Whereas the larger ($10,000) delayed reward was discounted less steeply than the smaller ($500) delayed reward, with probabilistic rewards the smaller reward was the one that was discounted less steeply.

Whereas the findings with respect to the delayed rewards are consistent with those of a number of previous studies (e.g., Benzion, Rapaport, & Yagil, 1989; Green et al., 1994, 1997; Kirby & Marakovic, 1996; Raineri & Rachlin, 1993), the findings with respect to probabilistic rewards are considerably more novel. In fact, to our knowledge, only one previous study has compared discounting of probabilistic rewards of different amounts. The results of a study by Ostaszewski et al. (1998) revealed that a larger probabilistic amount was not discounted at a lower rate than a smaller amount. The present experiment is the first to demonstrate
that, in fact, amount has opposite effects on the discounting of delayed and probabilistic rewards. Our finding that temporal and probability discounting can be dissociated by variations in the amount of delayed and probabilistic rewards poses problems for a single-process view of choice and decision making.

Experiment 2

The results of Experiment 1 called into question a fundamental assumption of contemporary choice theories, namely that a single process underlies the discounting of both delayed and probabilistic rewards (e.g., Myerson & Green, 1995; Rachlin, Logue, Gibbon, & Frankel, 1986; Rachlin et al., 1991). However, because Experiment 1 examined only two amounts of reward, it seemed prudent to attempt a systematic replication of these findings before making any strong theoretical claims. Moreover, previous research on discounting of delayed rewards has demonstrated the benefits of examining the effects of reward magnitude over an extended range. Green et al. (1997) found that rate of discounting decreased as the amount was increased up to around $10,000 to $25,000 and showed little further change between $25,000 and $100,000. It is not known whether the rate at which probabilistic rewards are discounted also approaches some asymptotic level. Accordingly, in Experiment 2, we examined the effects of reward magnitude on discounting rate over a wide range of amounts.

Method

Participants. Thirty students in the Faculty of Psychology at the University of Warsaw (Poland) participated in this experiment. There were 18 female and 12 male participants, ranging in age from 18 to 33 years ($M = 24.5$, $SD = 2.79$).

Procedure. The procedure was the same as in Experiment 1, except that the amounts of reward were different. Three reward amounts were used in Experiment 2: $200, $5,000, and $100,000. As in Experiment 1, each participant was studied with each amount presented at each of the six delays (1 month, 6 months, 1 year, 3 years, 5 years, and 10 years) and at each of the six probabilities (5%, 10%, 40%, 70%, 90%, and 95% chance), and with the 24 immediate, certain rewards presented in both increasing and decreasing order.

Results

As before, the subjective values were expressed as proportions so that the discounting of the different amounts of reward could be directly compared, and multiple nonlinear regression analysis was used to determine whether the discounting rate parameters (i.e., $k$ and $h$ for delayed and probabilistic rewards, respectively) differed as a function of amount. All three reward amounts were fit simultaneously with separate $k$ parameters and a single $s$ parameter.

Figure 3 shows the median subjective values of the $200 (circles), $5,000 (squares), and $100,000 (triangles) rewards and the best fitting discounting function for each delayed and probabilistic reward amount. (The inset shows a more detailed view of the data when the odds against receiving a reward were low.) Hyperbola-like functions (Equations 1 and 3) accurately described the discounting of both delayed and probabilistic rewards, and in both cases, the value of the $s$ parameter was significantly less than 1.0: for delayed rewards, $s = 0.71, t(14) = 3.55, p < .01$; for probabilistic rewards, $s = 0.60, t(14) = 5.70, p < .01$, supporting the inclusion of an exponent in both discounting functions. For the delayed rewards (top panel), Equation 1 accounted for 98.4% of the variance when fit simultaneously to the subjective values of the $200, $5,000, and $100,000 amounts. For the probabilistic rewards (bottom panel), Equation 3 accounted for 98.1% of the variance.

As may be seen in the top panel of Figure 3, when the rewards were delayed, the data points representing the subjective value of the smallest reward tended to fall below those representing the subjective value of the two larger rewards. This difference was reflected in the estimates for the discounting rate parameters in Equation 1. The value of $k$...
for $200 was 0.313, whereas the values of $k$ for $5,000 and $100,000 were 0.111 and 0.100, respectively. The $k$ for $200 differed significantly from the $k$s for the two larger amounts: $t(9) = 3.42, p < .01$, for $200 versus $5,000; $t(9) = 3.50, p < .01$, for $200 versus $100,000. The $k$ values for the two larger amounts did not differ significantly, $t(9) < 1.0$.

In contrast to the results obtained with delayed rewards, when the rewards were probabilistic, the data points representing the subjective value of the smallest reward tended to fall above those representing the subjective value of the larger rewards. This difference was reflected in the estimates for the discounting rate parameters in Equation 3. The values of $h$ for the $200, $5,000, and $100,000 rewards were 5.42, 8.23, and 14.31, respectively. The value of $h$ for $200 was significantly less than the value of $h$ for $5,000, $h(9) = 5.66, p < .01$, and the value of $h$ for $5,000 was significantly less than the value of $h$ for $100,000, $h(9) = 2.95, p < .05$, indicating that in each case the smaller reward was discounted at a significantly lower rate than the larger reward.

To evaluate whether the same pattern of results held at the level of the individual, we fit Equations 1 and 3 to the data from each individual participant. For delayed rewards, there was a significant effect of amount on the $k$ parameter in Equation 1 (Friedman’s repeated measures analysis of variance on ranks, $\chi^2 = 22.5, p < .0001$). Planned comparisons (Student Newman–Keuls) revealed that the individual $k$s for the smallest amount differed significantly from the $k$s for the two larger amounts (both $ps < .05$), which did not differ significantly from each other. For probabilistic rewards, there was a significant effect of amount on the $h$ parameter in Equation 3 (Friedman’s repeated measures analysis of variance on ranks, $\chi^2 = 22.5, p < .0001$). Planned pairwise comparisons (Student Newman–Keuls) revealed that the individual $h$s for the smallest amount differed significantly from the $h$s for the two larger amounts, which, in turn, differed significantly from each other (all $ps < .05$).

For the delayed rewards, 24 of the 30 participants had larger $k$ values for the smallest amount than for the largest amount, whereas for the probabilistic rewards, 28 of the participants showed the opposite pattern. That is, they had smaller $h$ values for the smallest than for the largest amount. Notably, for 22 of the 30 participants, the smallest amount was associated with both a larger $k$ and a smaller $h$ than the largest amount, significantly more than the number (i.e., 7.5) expected by chance (binomial test, $z = 5.90, p < .0001$). Finally, 21 participants had estimated values of the exponent, $s$, less than 1.0 for delayed rewards, and 23 had estimated values less than 1.0 for the probabilistic rewards, significantly more than expected by chance in both cases (binomial test, $z = 2.01, p < .05$, and $z = 2.74, p < .005$, respectively).

**Discussion**

Rate of discounting decreased as the amount of the delayed reward was increased, whereas the rate of discounting increased as the amount of the probabilistic reward was increased. These results replicate those of Experiment 1 and provide additional information regarding the range over which variation in amount of reward produces changes in discounting rate. For delayed rewards, the rate of discounting decreased as the amount was increased from $200 to $5,000 but showed no further decrease when the amount of the delayed reward was increased to $100,000. These results are consistent with those reported by Green et al. (1997). In contrast, for probabilistic rewards, the rate of discounting increased over the entire range examined in the present study. The difference in the range over which amount of reward affects the discounting of delayed and probabilistic rewards represents a further dissociation of the processes involved in the two cases. Despite this dissociation, the same form of hyperbola-like discounting function accurately described the data regardless of both the amount of the rewards and whether the rewards were delayed or probabilistic.

**General Discussion**

The same basic phenomena were observed in both experiments. On the one hand, large probabilistic amounts were discounted at higher rates than small amounts. On the other hand, large delayed amounts were discounted at lower rates than small amounts. Nevertheless, regardless of the amount of reward or whether it was delayed or probabilistic, discounting was well described by the same form of mathematical function.

**The Form of the Discounting Function for Delayed and Probabilistic Rewards**

A number of mathematical forms have been proposed to describe the decrease in subjective value of rewards with increases in the delay until their receipt. These forms include a simple hyperbola and an exponential decay, but the results of research comparing these two forms argue strongly against the exponential form as a model for the discounting of delayed rewards (e.g., Ainslie, 1992; Green & Myerson, 1993; Kirby, 1997; Mazur, 1987; Rachlin et al., 1991). In addition, Rachlin et al. (1991) have shown that the discounting of probabilistic rewards is better described by a simple hyperbola (Equation 2) than by an exponential.

More recently, the hyperbola-like function exemplified by Equations 1 and 3, in which the denominator of a hyperbola is raised to a power, has been shown to describe the discounting of delayed rewards better than a simple hyperbola without an exponent (Green et al., 1994; Myerson & Green, 1995). The present study extends this finding to discounting of probabilistic rewards, thereby raising two important questions: What is the significance of the specific form of the discounting function (i.e., what does the exponent mean), and what is the significance of the fact that the same form describes both types of decision making (i.e., that involving delayed rewards and that involving probabilistic rewards)?

With respect to the meaning of the exponent in the hyperbola-like discounting function, we have suggested that the exponent arises as a result of the nonlinear scaling of amount and time (Green et al., 1994; Myerson & Green, 1995). A similar suggestion was made by Rachlin (1989), who also noted the similarity between judgments in psychological experiments (Stevens, 1957) and judgments in
decision making involving rewards. Although psychophysical scaling may explain the exponent, it does not completely explain either the fact that the discounting functions for both delayed and probabilistic rewards are isomorphic or the particular hyperbola-like form of these functions.

The Effects of Amount on the Discounting of Delayed and Probabilistic Rewards

The fact that these discounting functions are isomorphic suggests that similar processes are involved in both kinds of decision making. This similarity has been explained in different ways by different theorists. One approach has been to explain one kind of decision making in terms of the other. For example, Rachlin (Rachlin et al., 1986, 1991) has suggested that decisions involving probabilistic rewards may be thought of as repeated gambles. In his view, probability is evaluated in terms of the expected waiting time until a win, and thus both kinds of decisions actually reflect the discounting of delayed rewards. Others (Benzion et al., 1989; Green & Myerson, 1996; Myerson & Green, 1995; Stevenson, 1986) have suggested that decisions involving delayed rewards may be thought of as gambles, in that waiting for rewards may be risky. In this view, delay is evaluated in terms of the expected odds against receiving the reward, and thus both kinds of decisions actually reflect the discounting of probabilistic rewards.

Although the present results are consistent with the idea that similar processes are involved in decisions about delayed and probabilistic rewards, they create difficulties for any attempt to reduce such decisions to a single process. If the discounting of delayed and probabilistic rewards were both the result of the same process, then one would expect that any variable that affects one type of decision making would also affect the other type of decision making in the same way. However, the present study demonstrated that the discounting of delayed and probabilistic rewards is affected by amount in opposite ways.

Figure 4 shows the rates of discounting for both delayed (top panel) and probabilistic (bottom panel) rewards observed in both experiments of the present study. Whereas small rewards were discounted more steeply than large rewards when outcomes were delayed, small rewards were discounted less steeply than large rewards when outcomes were uncertain (i.e., probabilistic). In addition, the rate at which probabilistic rewards were discounted increased continuously over the range of amounts studied, whereas there was a clear leveling off in the rate at which delayed rewards were discounted as amount increased. The latter result was also observed in a previous study (Green et al., 1997) that used delayed rewards. Thus, there is a range ($10,000 to $100,000 in the present study) over which amount appears to affect the discounting of probabilistic but not delayed rewards.

Further evidence that the same variable may not have the same effects on the discounting of probabilistic and delayed rewards was reported recently by Ostaszewski et al. (1998). These authors tested participants in Poland who were familiar with both Polish and U.S. currencies. The study took advantage of the fact that the Polish zloty had been associated with high rates of inflation whereas the U.S. dollar had been relatively stable. The results revealed that delayed rewards were discounted at higher rates when they were specified in zlotys than when they were specified in dollars. In contrast, probabilistic rewards specified in zlotys and probabilistic rewards specified in dollars were discounted at equivalent rates. These findings suggest that inflation affects monetary decisions involving future (i.e., delayed) rewards but does not affect decisions involving probabilistic rewards.2

\footnote{Ostaszewski et al.'s (1998) study consisted of three experiments. The third experiment, conducted in 1996, demonstrated that the differential effect of inflation on the discounting of delayed rewards observed in the first two experiments, which were conducted in 1994, disappeared after inflation was brought under control and a new currency was introduced. The experiments in the present study were conducted in 1997, and therefore the results presumably are not influenced by inflation.}
inflation on decisions involving delayed and probabilistic rewards, like the difference in the effects of amount observed in the present study, cannot easily be accommodated by a single-process theory.

**Risky Choice and Amount of Reward**

Perhaps the first single-process account of discounting is that from economics where preference is assumed to be based on the expected utility of an outcome. For probabilistic outcomes, expected utility depends on expected value, which is defined as the amount or nominal value of the outcome times the probability that it will occur. Application of this definition to probabilistic rewards is straightforward, but for delayed rewards it is necessary to assume that there is some risk associated with waiting. That is, it is assumed that probability decreases according to some specified mathematical function (e.g., the exponential decay function assumed by discounted utility theory).

As Prelec and Loewenstein (1991) pointed out, however, amount effects are anomalous from the economic perspective. Although a number of studies have shown such anomalous amount effects in the discounting of delayed rewards, there have been no previous experimental reports of anomalous amount effects with probabilistic rewards. Markowitz (1952) argued that individuals are risk-seeking for small rewards and risk-averse for large rewards, but the empirical basis for his position was an informal survey of his friends. Results of the present study, which represents the first systematic experiments on this issue and the first to show anomalous effects with both delayed and probabilistic rewards, are consistent with those of Markowitz.

To see how the present results demonstrate risk-seeking with small rewards and risk-aversion with large rewards, consider the relation of subjective value to expected value shown in Figure 5. As may be seen in the top panel, the function describing the relation between expected value and the odds against receiving a reward divides the space into two domains. When individuals' judgments of the subjective value of a reward exceed the reward's expected value, behavior is described as risk-seeking; when their judgment of the subjective value is less than the expected value, behavior is described as risk-averse.

As may be seen in the bottom panel, which presents the data for the largest and smallest amounts in Experiment 2, participants were less risk-averse for the smallest amount when the odds against receiving the amount were low (i.e., the subjective value of the smallest amount was closer to its expected value than was the subjective value of the largest amount). As the odds against receiving the probabilistic reward increased, participants were actually risk-seeking for the smallest amount, whereas they remained risk-averse for the largest amount (i.e., the subjective value of the smallest amount was greater than its expected value, whereas the subjective value of the largest amount was less than its expected value). A similar outcome was observed in Experiment 1, although because the large and small amounts differed less in magnitude than those used in Experiment 2, the effect was less pronounced.

![Figure 5](image-url)

**Figure 5.** Subjective value as a function of odds against receiving a reward. In the top panel, the broken line represents the change in the expected value (E.V.) of a reward as the odds against receiving it increase. As may be seen in the top panel, the expected value function divides the space into two domains: risk-seeking, when subjective value is greater than expected value, and risk-aversion, when subjective value is less than expected value. In the bottom panel, the group median subjective values for the largest and smallest amounts in Experiment 2 can be compared with their expected values, expressed as proportions. Note that when the odds against receiving a reward were greatest, participants were risk-seeking with respect to the smallest amount but risk-averse with respect to the largest amount.

At present, it is unclear why amount of reward has different effects on choices involving delayed and probabilistic rewards. One suggestion is that the way in which amount of reward affects discounting depends on whether the decision involves the possibility of a negative outcome. That is to say, choices involving a smaller, immediate reward and a larger, delayed reward involve deciding between two positive outcomes. However, choices involving a smaller, certain and a larger, probabilistic reward entail three possible outcomes: receiving the smaller reward, receiving the larger reward, or receiving nothing. Choosing to gamble on
the possibility of receiving the larger reward has been interpreted as involving the risk of disappointment if the gamble does not pay off (Bell, 1985; Prelec & Loewenstein, 1991).

Regardless of whether or not people experience the "anticipation of disappointment" (Prelec & Loewenstein, 1991, p. 782), it seems likely that the inclusion of a third possible outcome (i.e., receiving no reward) in the probabilistic choice situation is responsible for reversing the effect of amount that is observed with delayed rewards (where only positive outcomes are involved). Although people are typically risk averse when gambles are framed in terms of gains, the opposite result is observed (i.e., they tend to choose the riskier option) when the same gambles are framed in terms of losses (Tversky & Kahneman, 1981). In the current context, the possibility of receiving no reward may constitute a loss, leading to an amount effect opposite to that observed with choices involving delayed rewards in which both options are positive.

**Impulsivity and Self-Control**

The discounting of both delayed and probabilistic outcomes plays an important role in current theories of self-control (Ainslie, 1992; Logue, 1988; Green & Myerson, 1993; Rachlin, 1995), as well as in understanding a variety of psychological problems (e.g., addictions) that have been conceptualized in terms of impulsive behavior (Ainslie & Haendel, 1983; Herrnstein & Prelec, 1992; Heyman, 1996; Rachlin, 1990, 1992). Choosing a larger, more delayed reward over a smaller reward available sooner is generally assumed to reflect self-control, whereas the opposite choice is characterized as impulsive. Such "impulsive" decisions may result from the fact that the subjective value of a delayed reward is discounted, a large delayed reward may have a smaller subjective value than a more immediate, smaller reward. Thus, someone who is more impulsive than average may be characterized by steeper than average discounting of delayed rewards.

The term *impulsive* is also frequently used to describe risk-taking behavior. For example, impulsivity is a criterion for diagnosing borderline personality disorder (American Psychiatric Association, 1994), symptoms of which include a variety of high-risk behaviors (e.g., alcohol and drug abuse). Paradoxically, people who are impulsive may be characterized by lower than average rates of discounting probabilistic rewards, leading to a tendency to gamble on low-probability, positive outcomes, but by higher than average rates of discounting delayed rewards, leading to an inability to delay gratification and a tendency to disregard the delayed, negative consequences of their behavior.

The opposite effects of amount on the discounting of delayed and probabilistic rewards are in keeping with this paradoxical usage. Although steeper discounting of delayed rewards implies greater impulsivity and steeper discounting of probabilistic rewards implies greater self-control, increasing the amount of reward leads to more self-control in both contexts. For example, the present results suggest that decisions involving $1,000 will be more impulsive than decisions involving $100,000, regardless of whether these amounts are delayed or probabilistic, precisely because increasing the amount leads to less steep discounting of delayed rewards but more steep discounting of probabilistic rewards.

Thus, amount appears to have similar effects on what is frequently termed *impulsive behavior* regardless of whether choices involve delayed or probabilistic rewards. Similarly, development seems to have similar effects on both aspects of self-control, in that the ability to delay gratification increases with age from adolescence to adulthood at the same time as risk-taking decreases (e.g., Arnett, 1992). These changes are hard to understand from a single-process perspective. That is, if all decisions are viewed from the perspective of risk, then as one becomes less likely to choose a risky over a certain outcome, one should also become more likely to choose an immediate (hence more certain) over a delayed (hence riskier) outcome. Alternatively, if all decisions are viewed from the perspective of delay, then as one becomes less likely to choose an immediate over a delayed outcome, then one should become more likely to choose a risky outcome (one involving more repeated gambles and hence a longer delay before a win) over a certain (hence more immediate) outcome.

From the perspective of self-control, therefore, the opposite effects of amount on rates of discounting delayed and probabilistic rewards are consistent with the view that such decisions involve processes that, although not identical, are at least fundamentally similar in nature. Increases in amount lead to increases in what may be termed *self-control* in both contexts. That is, increases in amount lead to increases in the tendency to choose the delayed, larger reward over a smaller but more immediate reward, as well as to increases in the tendency to choose the smaller but more certain reward over a larger but riskier reward. This finding, together with the fact that the discounting functions have the same mathematical form, suggest that it may be possible to develop a *unified* account of decision making involving delayed and probabilistic rewards even if it is not possible to develop a *unitary* account.

**References**


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